

# Application of the Lorentz-transform technique to meson photoproduction

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Received: 21 February 2003 / Revised version: 13 April 2003 /  
Published online: 1 July 2003 – © Società Italiana di Fisica / Springer-Verlag 2003  
Communicated by U.-G. Meißner

**Abstract.** We show that the Lorentz integral transform (LIT) technique which has been successfully applied to photoreactions in light nuclei can also be applied to photoreactions involving particle production. A simple model where results are easily calculable in the traditional fashion is used to test the technique. Specifically, we compute inclusive  $\pi^+$  photoproduction from deuterium for photon energies less than 200 MeV using a Yamaguchi model for the  $NN$  interaction. It is demonstrated that, although the response functions for inclusive meson production do not have favourable asymptotic behavior, one can nonetheless extract them by inversion of the transform. The implication is that one can treat realistic problems of photo-meson production, including all final-state interactions, by means of the LIT technique.

**PACS.** 13.60.Le Meson production – 21.45.+v Few-body systems – 25.10.+s Nuclear reactions involving few-nucleon systems – 25.20.Lj Photoproduction reactions

## 1 Introduction

In a series of papers [1–7] it has been demonstrated that the LIT technique allows a convenient calculation of inclusive photoreaction cross-sections wherein final-state interactions are fully included. Further, the technique has been extended [8,9] to exclusive reactions. A merit of the technique is that the calculation of continuum wave functions is avoided. In fact the differential equations to be solved are inhomogeneous and have solutions bounded at infinity. The work cited above is based on non-relativistic quantum mechanics and a nucleons-only subspace.

In order to treat meson photoproduction from nuclei it is desirable to see if the LIT technique can be extended to a larger subspace, *i.e.* one including nucleons and mesons. This would enable the full inclusion of both meson-nucleon and  $NN$  interactions in the production calculations. As a test we consider inclusive photoproduction of low-energy ( $< 40$  MeV)  $\pi^+$ -mesons from deuterium. A traditional calculation of this process was reported by Dressler, MacDonald, and O’Connell [10], and by Gupta, Anand, and Bhasin [11]. Both of these groups employed a simple Yamaguchi potential [12] with parameters chosen to account

for deuteron binding and low-energy  $NN$  scattering properties. We adopt this model with the parameters used in [10]. Thus, this example attempts to apply the LIT technique in the  $NN\pi$  subspace, where, for simplicity, only an  $NN$  interaction is included. Although no  $\pi$ - $N$  scattering is contained in this model, this is not essential for testing the method. Our results will show that it is indeed possible to apply the LIT method to this problem and to extract the total photoproduction cross-section. We should point out that our restriction to a simple  $NN$  potential for this test calculation does not modify these conclusions. This is clearly seen from the series of papers [1–7] where a range of  $NN$  potentials, from simple phenomenological models to modern realistic potentials, have been used in 3-nucleon and 4-nucleon problems. In no case did the complexity of the  $NN$  potential model have any effect on the accuracy or implementation of the LIT technique. With assurance that the formalism works, the addition of a meson-nucleon interaction can be viewed as a technical point, the consequence of which would be more complicated numerics.

Low-energy photoproduction is described by the Kroll-Ruderman [13] operator

$$\mathcal{H}_{\text{int}}(x) = -ie \left( \frac{f}{m_\pi} \right) \times \sum_j \hat{\epsilon}_\lambda \cdot \vec{\sigma}(j) \tau_-(j) e^{-ik \cdot x} \phi_+(x) \delta(\vec{x} - \vec{x}_j), \quad (1)$$

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where  $\vec{k}$  and  $\hat{\epsilon}_\lambda$  denote the incident photon momentum and polarization vectors, respectively, and  $\phi_+(x)$  denotes the meson field operator

$$\phi_+(x) = \frac{1}{\sqrt{8\pi^3}} \times \int d^3q \frac{1}{\sqrt{2q_0}} \left[ e^{iq \cdot x} a_+^\dagger(\vec{q}) + e^{-iq \cdot x} a_-(\vec{q}) \right] \quad (2)$$

with  $q_0$  the energy of the meson and  $a^\dagger$  and  $a$  are the usual creation and annihilation operators. Here,  $e$  is the positive elementary charge,  $m_\pi$  is the  $\pi^+$ -meson mass,  $f$  is the  $\pi$ - $N$  coupling constant,  $\vec{\sigma}$  denotes the Pauli spin matrix vector and  $\tau_-$  the isospin operator. Dressler *et al.* [10] show that the KR term gives nearly the entire cross-section for pion energies in the range considered here. Due to cancellations, other terms in the production operator, *i.e.* terms required by gauge invariance, do not contribute significantly at low pion energies. With  $E_{\text{cm}}$  as the incident c.m. energy, the response function  $\mathcal{R}(E_{\text{cm}})$  for this process is

$$\mathcal{R}(E_{\text{cm}}) = \frac{1}{6} \sum_{M_d, \lambda} \sum_f \left| \langle f | \tilde{\mathcal{O}}(\vec{k}, \lambda) | D, M_d \rangle \right|^2 \times \delta(E_{\text{cm}} - E_f), \quad (3)$$

where  $|D, M_d\rangle$  denotes the deuteron ground state with polarization  $M_d$  and

$$\tilde{\mathcal{O}}(\vec{k}, \lambda) = \int d^3q \mathcal{O}(\vec{k}, \lambda, \vec{q}) a_+^\dagger(\vec{q}) \quad (4)$$

with

$$\mathcal{O}(\vec{k}, \lambda, \vec{q}) = -ie \left( \frac{f}{m_\pi} \right) \frac{1}{\sqrt{8\pi^3}} \frac{1}{\sqrt{2q_0}} \times \sum_j \hat{\epsilon}_\lambda \cdot \vec{\sigma}(j) \tau_-(j) e^{i\vec{x}_j \cdot (\vec{k} - \vec{q})}. \quad (5)$$

In eq. (3) above  $|f\rangle$  denotes the wave function of the relative motion of the  $nn\pi^+$  system with energy  $E_f$ , and  $\sum_f$  indicates an integration over all relative momenta and a sum over all final nucleon spins. The inclusive cross-section is related to  $\mathcal{R}(E_{\text{cm}})$  by

$$\sigma(E_{\text{cm}}) = \frac{2\pi^2}{k} \mathcal{R}(E_{\text{cm}}). \quad (6)$$

Relative momenta in the final state are taken as

$$\vec{p}_x = (\vec{p}_1 - \vec{p}_2)/2, \quad (7)$$

$$\vec{p}_y = -\frac{m_\pi}{M} (\vec{p}_1 + \vec{p}_2) + \frac{2m_n}{M} \vec{p}_\pi, \quad (8)$$

where  $\vec{p}_1$ ,  $\vec{p}_2$ , and  $\vec{p}_\pi$  are the momenta of the final-state neutrons and pion, respectively, and  $M$  is the total mass  $2m_n + m_\pi$  with the mass of the neutron  $m_n$ . As implied by this separation, we are restricting ourselves to non-relativistic kinematics for both the nucleons and the pion. This allows the separation of the Hamiltonian into Jacobi

coordinates for the  $NN\pi$  three-body system. In terms of these quantities, the energy-conserving  $\delta$ -function appearing in  $\mathcal{R}(E_{\text{cm}})$  takes the detailed form

$$\delta(E_{\text{cm}} - E_f) = \delta \left( E_{\text{cm}} - (2m_n + m_\pi) - \frac{p_x^2}{m_n} - \frac{p_y^2}{2\mu} \right), \quad (9)$$

where  $\mu$  is the reduced two-neutron-pion mass. Finally, it is more convenient to use the c.m. energy above threshold

$$\mathcal{W} = k + \frac{k^2}{2m_d} - (2m_n + m_\pi - m_d) \geq 0,$$

where  $m_d$  is the mass of the deuteron. The LIT, referred to hereafter as the transform, of the response function  $\mathcal{R}(\mathcal{W})$  is then defined with  $\sigma_R, \sigma_I > 0$  as

$$\begin{aligned} L(\sigma_R, \sigma_I) &= \int_0^\infty d\mathcal{W} \frac{\mathcal{R}(\mathcal{W})}{(\mathcal{W} - \sigma_R)^2 + \sigma_I^2} \quad (10) \\ &= \frac{1}{6} \sum_{M_d, \lambda} \int_0^\infty d\mathcal{W} \langle D, M_d | \tilde{\mathcal{O}}^\dagger(\vec{k}, \lambda) \\ &\quad \times \frac{\delta(\mathcal{W} - H)}{(H - \sigma_R)^2 + \sigma_I^2} \tilde{\mathcal{O}}(\vec{k}, \lambda) | D, M_d \rangle. \quad (11) \end{aligned}$$

Because of the relation of  $k$  to  $\mathcal{W}$ , a straightforward integration of the above equation would leave the operator  $\tilde{\mathcal{O}}(\vec{k}, \lambda)$  depending in a complicated way on the Hamiltonian  $H$ . Therefore, we proceed by setting  $\vec{k}$  appearing in the operator  $\tilde{\mathcal{O}}$  to a constant arbitrarily chosen ‘‘pseudo-momentum’’  $\vec{k}_p$ . As a result, we introduce a new transform,  $L_{k_p}$ , which depends on  $k_p$  and takes the form

$$\begin{aligned} L_{k_p}(\sigma_R, \sigma_I) &= \frac{1}{6} \sum_{M_d, \lambda} \langle D, M_d | \tilde{\mathcal{O}}^\dagger(\vec{k}_p, \lambda) \\ &\quad \times \frac{1}{(H - \sigma_R)^2 + \sigma_I^2} \tilde{\mathcal{O}}(\vec{k}_p, \lambda) | D, M_d \rangle \quad (12) \end{aligned}$$

$$= \frac{1}{6} \sum_{M_d, \lambda} \langle \tilde{\psi}_{M_d, \lambda}(\sigma_R, \sigma_I, k_p) | \tilde{\psi}_{M_d, \lambda}(\sigma_R, \sigma_I, k_p) \rangle, \quad (13)$$

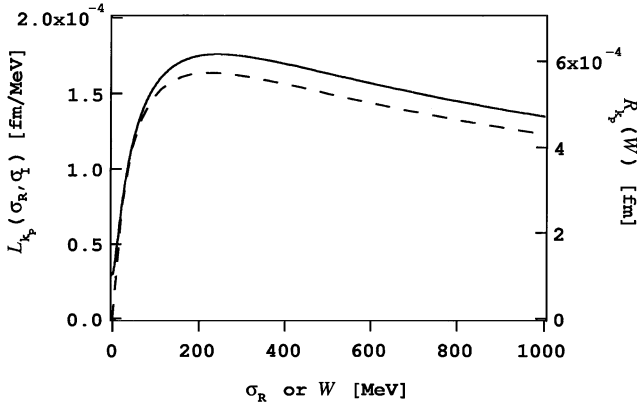
where the Lorentz function  $\tilde{\psi}$  is solution of the inhomogeneous equation

$$(H - \sigma_R + i\sigma_I) | \tilde{\psi}_{M_d, \lambda}(\sigma_R, \sigma_I, k_p) \rangle = \tilde{\mathcal{O}}(\vec{k}_p, \lambda) | D, M_d \rangle \quad (14)$$

with

$$\begin{aligned} \langle \vec{p}_x, \vec{p}_y | H | \vec{p}'_x, \vec{p}'_y \rangle = \\ \left[ \left( \frac{p_x^2}{m_n} + \frac{p_y^2}{2\mu} \right) \delta(\vec{p}_x - \vec{p}'_x) + V(\vec{p}_x, \vec{p}'_x) \right] \delta(\vec{p}_y - \vec{p}'_y). \quad (15) \end{aligned}$$

The inverse to the transform  $L_{k_p}(\sigma_R, \sigma_I)$  will lead to a corresponding response function  $\mathcal{R}_{k_p}(\mathcal{W})$  and in turn will yield the correct cross-section for  $k_p = k$ . Consequently, for each photon energy the calculation of the transform and its inversion must be repeated.



**Fig. 1.**  $L_{k_p}(\sigma_R, \sigma_I)$  (solid line) and  $\mathcal{R}_{k_p}(\mathcal{W})$  (dashed line) shown for  $\sigma_I = k_p = 10$  MeV.

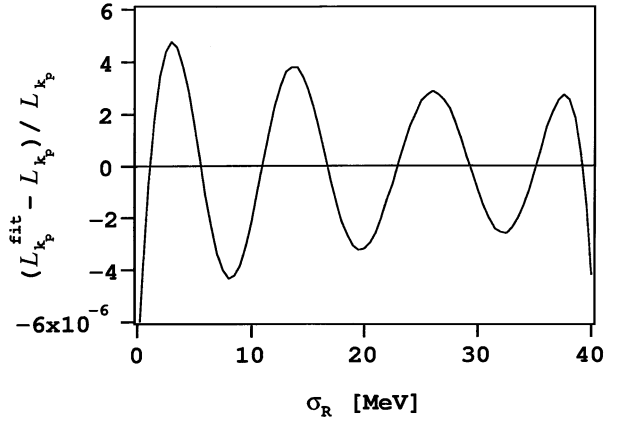
As mentioned earlier, the simple model here only includes an  $NN$  potential, *i.e.*,  $V(\vec{p}_x, \vec{p}_x')$ . The kinetic energy of the meson with respect to the  $nn$  pair appears in  $H$  as the term proportional to  $p_y^2$ . Meson rescattering could be included by adding to  $H$  appropriate potentials. Although that would considerably complicate the numerical aspect, it would not affect the question posed here, namely: can the LIT be inverted when the response function arises from a particle production process? A problem is the slow fall-off of the response function for high  $\mathcal{W}$  values. From the definition of the transform it is clear that the response must behave asymptotically like  $\mathcal{W}^{1-x}$ , where  $x > 0$  in order for the integral to converge. In the case of nuclear photoabsorption without pion production, the response function falls off very rapidly for increasing  $\mathcal{W}$  resulting in  $L(\sigma_R, \sigma_I)$  also falling rapidly for large  $\sigma_R$ . Inversion then gives an accurate account of  $\mathcal{R}(\mathcal{W})$  over its entire range by calculating  $L(\sigma_R, \sigma_I)$  for a finite number of  $\sigma_R$  values. That the response function for inclusive photoproduction may have a significantly different asymptotic behavior can be seen from the basic Kroll-Rudermann [13] cross-section for  $p(\gamma, \pi^+)n$ :

$$\sigma = 2\alpha \left( \frac{f}{m_\pi} \right) \left( \frac{q}{k} \right) \frac{E_n(q) E_p(k)}{E_{\text{cm}}^2}, \quad (16)$$

where  $E_N$  are the nucleon energies,  $E_{\text{cm}}$  is the c.m. energy  $k + E_p(k)$  and  $\alpha$  the fine-structure constant. This cross-section approaches a constant for large  $E_{\text{cm}}$  and the response  $\mathcal{R}(E_{\text{cm}})$  rises linearly in  $E_{\text{cm}}$ . In the next section it will be seen that the pion production response function for a finite nucleus is tempered at large  $E_{\text{cm}}$  by structure effects. Nevertheless it still rises over a large energy region thereby requiring a different approach for the inversion of the transform.

## 2 Results

Details of the model and the parameters used are given in the appendix. There, it is seen that the vector  $|\tilde{\psi}_{M_d, \lambda}(\sigma_R, \sigma_I, k_p)\rangle$  can also be labeled by the final



**Fig. 2.** Relative error of the fit  $L_{k_p}^{\text{fit}}(\sigma_R, \sigma_I)$  compared to the computed transform  $L_{k_p}(\sigma_R, \sigma_I)$ . Here we have used  $\sigma_I = k_p = 10$  MeV for illustration.

neutron-neutron spin and that the response function is therefore a sum of singlet and triplet contributions. The separable potential used here only has scattering in the spin  $S = 0$ , isospin  $T = 1$  channel, while the  $S = 1$ ,  $T = 1$  final state consisting of odd partial waves is non-interacting. Figure 1 shows the transform  $L_{k_p}(\sigma_R, \sigma_I)$  for the case  $\sigma_I = k_p = 10$  MeV. One notes that  $L_{k_p}$  reaches a maximum at  $\sigma_R \approx 200$  MeV and then falls very slowly with increasing  $\sigma_R$ . Also shown in this figure is the response function  $\mathcal{R}_{k_p}(\mathcal{W})$  for  $k_p = 10$  MeV. The response function behaves similarly to  $L$  as one would expect, since  $L(\sigma_R, \sigma_I)$  samples  $\mathcal{R}$  mainly from a region centered at  $\mathcal{W} = \sigma_R$ . We note that  $\mathcal{R}_{k_p}(\mathcal{W})$  does not rise linearly with energy as in the case of  $p(\gamma, \pi^+)n$  but falls off slowly with energy after reaching a maximum. Unfortunately, the fall-off of  $L$  with  $\sigma_R$  is so slow and covers such a large energy range that inverting it to obtain  $\mathcal{R}(\mathcal{W})$  over the full range, as was possible in the earlier photoabsorption calculations, is not only very difficult but largely physically meaningless, because our model is only valid for non-relativistic mesons. In fact the present model is only sensible for energies  $\mathcal{W}$  not exceeding approximately 40 MeV, which corresponds to the maximal pion energy still being non-relativistic. Our aim then is to calculate  $L$  only for the segment  $0 \leq \sigma_R \leq 40$  MeV and then invert it to extract  $\mathcal{R}_{k_p}(\mathcal{W})$  for  $\mathcal{W}$  in the same energy range. Since  $\mathcal{R}$  can be calculated directly, the error can be easily assessed.

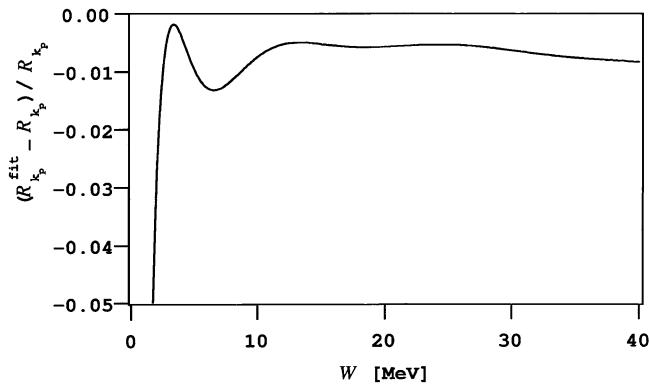
The inversion process we use has already been described in [3] but a brief account is as follows. The response function  $\mathcal{R}_{k_p}(\mathcal{W})$  is written as a sum:

$$\mathcal{R}_{k_p}(\mathcal{W}) = \sum_{i=1}^N \alpha_i \mathcal{W}^{S+2} e^{-\beta_i \mathcal{W}} \equiv \sum_{i=1}^N \mathcal{R}_{k_p}^i(\mathcal{W}) \quad (17)$$

and the parameters  $\alpha_i$  and  $\beta_i$  are determined by fitting

$$L_{k_p}^{\text{fit}}(\sigma_R, \sigma_I) = \sum_{i=1}^N \int_0^\infty d\mathcal{W} \frac{\mathcal{R}_{k_p}^i(\mathcal{W})}{(\mathcal{W} - \sigma_R)^2 + \sigma_I^2} \quad (18)$$

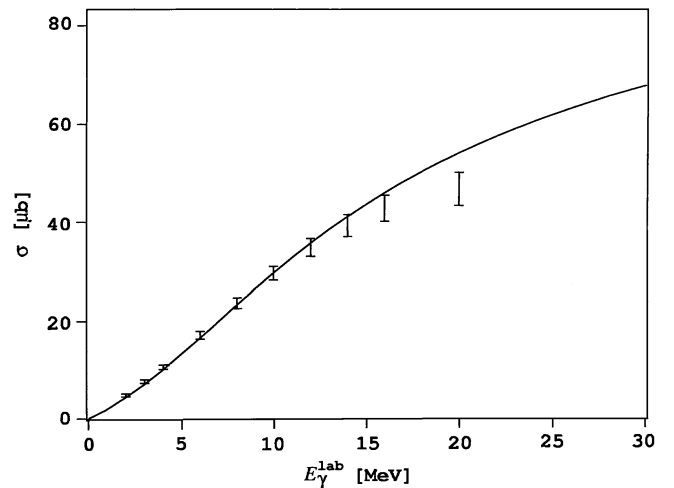
to  $L$  as computed from eqs. (A.12), (A.13) of the appendix. Note that the threshold behavior  $\mathcal{W}^{S+2}$ , where



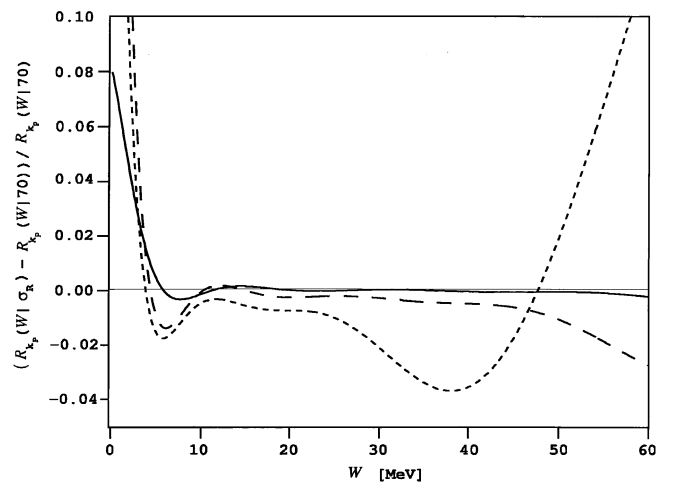
**Fig. 3.** Relative error between the response function  $\mathcal{R}_{k_p}^{\text{fit}}(\mathcal{W})$  and  $\mathcal{R}_{k_p}(\mathcal{W})$  computed in the traditional method for  $\sigma_I = k_p = 10$  MeV.

$S$  is the total spin, is appropriate to the photoproduction process [11] and differs slightly from the form used in the earlier photoabsorption calculations. It turns out, however, that the quality of the fit is nearly independent of which threshold behavior is used. Figure 2 shows the quality of the fit obtained using  $N = 10$ . With the parameters so determined one can calculate the response functions and finally the inclusive cross-section. As mentioned earlier, this model problem is simple enough that the response functions are easily calculated in the traditional manner [10,11]. Figure 3 shows the relative error between the response function as calculated from the LIT technique ( $\mathcal{R}_{\text{fit}}$ ) and the one calculated in the traditional manner. Over most of the energy range between  $0 \leq \mathcal{W} \leq 40$  MeV, the relative error is in the 1% range. However, near the threshold, *i.e.*  $\mathcal{W} < 2$  MeV, the response function tends to zero for  $\mathcal{W} \rightarrow 0$  and limited numerical accuracy produces an exaggerated relative error in this region. These effects are too small to be visible in a plot of the cross-section however. Finally, our computed cross-section along with data of Booth *et al.* [14] is shown in fig. 4. The results of using the LIT technique are indistinguishable from the traditional methods in [10,11].

It is instructive to see the differences in the response functions that occur if one uses transforms calculated in different ranges of  $\sigma_R$ . One expects that, because of the nature of the Lorentz transform, one should only have to calculate the transform up to  $\sigma_R = 40$  MeV, as was done above, if the response function was only required in the energy range  $0 \leq \mathcal{W} \leq 40$  MeV. To show this, we calculate the transform in four ranges,  $0 \leq \sigma_R \leq \sigma_{R_{\text{max}}}$ , where  $\sigma_{R_{\text{max}}} = 20, 40, 60,$  and  $70$  MeV. The respective response functions obtained by inverting each of these transforms is denoted by  $\mathcal{R}_{k_p}(\mathcal{W}|\sigma_{R_{\text{max}}})$ . Figure 5 shows the error in the  $\sigma_{R_{\text{max}}} = 20, 40, 60$  MeV cases relative to the  $\sigma_{R_{\text{max}}} = 70$  MeV case. One notes that the  $\sigma_{R_{\text{max}}} = 20$  MeV case only is accurate for energies up to 20 MeV, that the  $\sigma_{R_{\text{max}}} = 40$  MeV case has a less than 1% error at  $\mathcal{W} = 40$  MeV, and that the  $\sigma_{R_{\text{max}}} = 60$  MeV case has only a very small error up to 60 MeV.



**Fig. 4.** Calculated total cross-section for  $D(\gamma, \pi^+)nn$  shown together with the data of [14]. The cross-sections calculated either by the LIT technique or the traditional method [10,11] are indistinguishable.



**Fig. 5.** Relative errors in response functions for  $\sigma_I = k_p = 10$  MeV with respect to the  $\sigma_{R_{\text{max}}} = 70$  MeV case:  $\sigma_{R_{\text{max}}} = 60$  MeV (solid line),  $\sigma_{R_{\text{max}}} = 40$  MeV (long-dashed line),  $\sigma_{R_{\text{max}}} = 20$  MeV (short-dashed line).

### 3 Conclusions

We have shown that the LIT technique can be used to calculate inclusive meson photoproduction cross-sections in the non-relativistic regime. Rather than fitting the transform over its entire range, as was possible in earlier photoabsorption calculations, one fits here only the low-energy segment to obtain the low-energy response functions. The model calculation used here shows that the response functions thus obtained are as accurate as numerical techniques will allow. Our next step will be to add a pion-nucleon interaction to the Hamiltonian in order to take account of pion scattering effects. The dynamical model of Darwish, Arenhoevel, and Schwamb [15] would

provide a benchmark calculation against which to further check our method. It would also be of interest to apply these types of calculations to higher- $A$  nuclei such as  $^3\text{He}$ ,  $^3\text{H}$ , or  $^4\text{He}$ . The LIT technique extends readily to these cases as well as being able to handle realistic potentials including Coulomb effects. One should expect to be able to study meson rescattering effects with realistic nuclear models in a variety of light nuclei.

We would like to thank H. Arenhövel for valuable comments. All authors acknowledge support from the Italian Ministry of Research (MURST). In addition, the work of C.R. and E.L.T. is supported by the National Science and Engineering Research Council of Canada.

## Appendix A.

For the  $NN$  interaction we use the separable model of Y. Yamaguchi [12], which is a pure  $S$ -wave interaction:

$$V(\vec{p}, \vec{p}') = -\lambda_0 g_0(\vec{p}) g_0(\vec{p}') \frac{1}{4} (1 - \vec{\sigma}_1 \cdot \vec{\sigma}_2) - \lambda_1 g_1(\vec{p}) g_1(\vec{p}') \frac{1}{4} (3 + \vec{\sigma}_1 \cdot \vec{\sigma}_2), \quad (\text{A.1})$$

$$g_S(\vec{p}) = \frac{1}{\vec{p}^2 + \beta_S^2}, \quad S \in \{0, 1\}. \quad (\text{A.2})$$

Here, the labels 0 and 1 refer to the spin-singlet and spin-triplet parts, respectively. The following more up-to-date constants for this model have been taken from [10]:

$$\alpha = 0.2316 \text{ fm}^{-1}, \quad \beta_0 = 1.129 \text{ fm}^{-1}, \quad \beta_1 = 1.392 \text{ fm}^{-1}, \quad (\text{A.3})$$

$$\lambda_0 = 0.02774 \text{ fm}^{-2}, \quad \lambda_1 = \frac{\beta_1 (\alpha + \beta_1)^2}{m_n \pi^2}. \quad (\text{A.4})$$

These constants fit the deuteron binding energy, the experimental values for the singlet and triplet scattering lengths and the singlet effective range.

Using this separable  $NN$  interaction, one obtains for the deuteron ground state

$$\psi_d(\vec{p}) = \frac{\sqrt{\alpha \beta_1 (\alpha + \beta_1)^3}}{\pi} \frac{1}{(\vec{p}^2 + \alpha^2) (\vec{p}^2 + \beta_1^2)} \quad (\text{A.5})$$

and a binding energy of 2.224 MeV.

The Lorentz function  $\tilde{\psi}_{M_d, \lambda}^S(\sigma, k_p, \vec{p}_x, \vec{p}_y)$  can be decomposed into its spin components as

$$\tilde{\psi}_{M_d, \lambda}^S(\sigma, k_p, \vec{p}_x, \vec{p}_y) = \sum_{S=0,1} \tilde{\psi}_{M_S}^S(\sigma, k_p, \vec{p}_x, \vec{p}_y), \quad (\text{A.6})$$

where  $M_S = M_d + \lambda$  with  $M_d$  and  $\lambda$  the deuteron and photon polarization, respectively, and the function

$\tilde{\psi}_{M_S}^S(\sigma, k_p, \vec{p}_x, \vec{p}_y)$  is a solution of

$$\left[ \frac{p_x^2}{m_n} + \frac{p_y^2}{2\mu} - \sigma \right] \tilde{\psi}_{M_S}^S(\sigma, k_p, \vec{p}_x, \vec{p}_y) - \lambda_S g_S(p_x) C_{M_S}^S(\vec{p}_y, \vec{\Delta}) = \Xi_{M_S, M_d, \lambda}^S \mathcal{F}_{\vec{\Delta}}^S(\vec{p}_x), \quad S \in \{0, 1\}, \quad (\text{A.7})$$

where

$$C_{M_S}^S(\vec{p}_y, \vec{\Delta}) = \int d^3p g_S(p) \tilde{\psi}_{M_S}^S(\sigma, k_p, \vec{p}, \vec{p}_y), \quad (\text{A.8})$$

$$\mathcal{F}_{\vec{\Delta}}^S(\vec{p}) = \psi_d(\vec{p} - \vec{\Delta}) + (-1)^S \psi_d(\vec{p} + \vec{\Delta}), \quad (\text{A.9})$$

$$\Xi_{M_S, M_d, \lambda}^S = -i \frac{3\sqrt{2}ef}{m_\pi} (-1)^{1+S-M_S} \sqrt{2S+1} \times \begin{pmatrix} S & 1 & 1 \\ -M_S & \lambda & M_d \end{pmatrix} \begin{Bmatrix} \frac{1}{2} & S & \frac{1}{2} \\ 1 & \frac{1}{2} & 1 \end{Bmatrix}. \quad (\text{A.10})$$

We take  $f^2/4\pi = 0.078$ .

The constant  $C_{M_S}^S(\vec{p}_y, \vec{\Delta})$  with respect to  $p_x$  is

$$C_{M_S}^S(\vec{p}_y, \vec{\Delta}) = m_n \Xi_{M_S, M_d, \lambda}^S \int d^3p \frac{g_S(p) \mathcal{F}_{\vec{\Delta}}^S(\vec{p})}{p^2 + \gamma^2 - m_n \sigma} \times \left[ 1 - \frac{m_n \pi^2 \lambda_S}{\beta_S (\beta_S + \sqrt{\gamma^2 - m_n \sigma})^2} \right]^{-1} \quad (\text{A.11})$$

with  $(m_n/\mu)/2\vec{p}_y^2 \equiv \gamma^2(\vec{p}_y)$ . In the case of  $S = 1$  this constant vanishes.

The solution of the Lorentz equation therefore is

$$\tilde{\psi}_{M_S}^S(\sigma, k_p, \vec{p}_x, \vec{p}_y) = \frac{m_n \Xi_{M_S, M_d, \lambda}^S \mathcal{F}_{\vec{\Delta}}^S(\vec{p}_x) + m_n \lambda_S g_S(p_x) C_{M_S}^S(\vec{p}_y, \vec{\Delta})}{p_x^2 + \gamma^2 - m_n \sigma}, \quad (\text{A.12})$$

and the transform  $L_{k_p}(\sigma_R, \sigma_I)$  is

$$L_{k_p}(\sigma_R, \sigma_I) = \frac{1}{6} \sum_{S, M_d, \lambda} \langle \tilde{\psi}_{M_S}^S | \tilde{\psi}_{M_S}^S \rangle. \quad (\text{A.13})$$

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